14 Topics in Concurrency (GW)

This question is on basic Petri nets (Petri nets in which the pre- and post-condition multirelations are relations and all places have capacity 1) and the modal μ-calculus.

(*a*) Draw basic Petri nets to illustrate each of the following:

(*i*) Independence (also called concurrency)

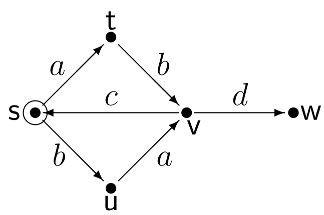
(*ii*) Backwards conflict

(*iii*) Forwards conflict

(*iv*) Contact [4 marks]

(*b*) Prove that if $\mathcal{M} \xrightarrow{e_1} \mathcal{M}_1 \xrightarrow{e_2} \mathcal{M}'$ and e_1 and e_2 are independent events in a basic Petri net then there exists a marking $\mathcal{M}_2$ such that $\mathcal{M} \xrightarrow{e_2} \mathcal{M}_2 \xrightarrow{e_1} \mathcal{M}'$.
[3 marks]

(*c*) Draw a basic Petri net with four events $\{a, b, c, d\}$ that gives rise to the following transition system. The states of the transition system should correspond to reachable markings of the Petri net, the state corresponding to the initial marking should be s, and the transitions should be labelled by the event of the Petri net that generates them.



[3 marks]

(*d*) With respect to the transition system drawn in part (*c*), determine which of the states $\{\mathsf{s}, \mathsf{t}, \mathsf{u}, \mathsf{v}, \mathsf{w}\}$ satisfy the following modal μ-calculus assertions:

(*i*) $\mu X.(\langle d\rangle T \vee \langle .\rangle X)$

(*ii*) $\nu Y.([.]Y \wedge \langle .\rangle T)$

Justify your answers. [4 marks]

(*e*) Prove that in a finite-state transition system, $s \models \nu X.(\langle a\rangle T \wedge \langle .\rangle X)$ if and only if there exists an infinite path from s along which an a-action can occur in every visited state. [6 marks]