## 9 Information Theory and Coding (JGD)

(*a*) Consider an alphabet of 5 symbols whose probabilities are as follows:

| A | B | C | D | E |
|---|---|---|---|---|
| $\frac{1}{16}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{16}$ | $\frac{1}{2}$ |

One of these symbols has been selected at random and you need to discover which symbol it is by asking 'yes/no' questions that will be truthfully answered.

(*i*) What would be the most efficient sequence of such questions that you could ask in order to discover the selected symbol? [2 marks]

(*ii*) By what principle can you claim that each of your proposed questions in the sequence is maximally informative? [2 marks]

(*iii*) On average, how many such questions will need to be asked before the symbol is discovered? What is the entropy of the symbol set? [2 marks]

(*iv*) Construct a uniquely decodable prefix code for the symbols. Explain why it is uniquely decodable and why it has the prefix property. [2 marks]

(*v*) Relate the bits in the code words forming your prefix code to the 'yes/no' questions that you proposed in (*i*). [2 marks]

(*b*) Explain how the bits in an IrisCode are set by phase sequencing. Discuss how quantisation of the complex plane into phase quadrants sets each pair of bits; why it is beneficial for quadrant codes to form a Gray Code; how much entropy is thereby typically extracted from iris images; and why such bit sequences enable extremely efficient identity searches and matching. [5 marks]

(*c*) Consider a noisy analog communication channel of bandwidth $\omega = 1$ MHz, which is perturbed by additive white Gaussian noise whose total spectral power is $N_0\omega = 1$. Continuous signals are transmitted across such a channel, with average transmitted power $P = 1{,}000$. Give a numerical estimate for the *channel capacity*, in bits per second, of this noisy channel. Then, for a channel having the same bandwidth $\omega$ but whose signal-to-noise ratio $\frac{P}{N_0\omega}$ is four times better, repeat your numerical estimate of capacity in bits per second. [5 marks]