

6 Denotational Semantics (MPF)

- (a) Let D be a poset and let $f : D \rightarrow D$ be a monotone function.
- (i) Give the definition of the least pre-fixed point, $fix(f)$, of f . Show that $fix(f)$ is a fixed point of f . [5 marks]
 - (ii) Show that whenever D is a domain and f is a continuous function, $fix(f)$ exists. [5 marks]
- (b) A poset (P, \sqsubseteq) has *binary meets* if for every pair of elements $x, y \in P$ there is a necessarily unique element $(x \sqcap y) \in P$ such that
- $(x \sqcap y) \sqsubseteq x$ and $(x \sqcap y) \sqsubseteq y$, and
 - for all $z \in P$, $z \sqsubseteq x$ and $z \sqsubseteq y$ imply $z \sqsubseteq (x \sqcap y)$.
- (i) Let (P, \sqsubseteq) be a poset with binary meets. Show that the function $meet : P \times P \rightarrow P$ given by $meet(x, y) = x \sqcap y$ is monotone. [5 marks]
- (ii) Exhibit a domain with binary meets for which the function $meet$ is not continuous. Justify your answer. [5 marks]