1 Advanced Graphics (NAD)

(*a*) A general piecewise curve definition, whether Bézier, B-spline, or NURBS, can be written as a sum of products of basis functions, $A_i(t)$, and control points, $\mathbf{P}_i$:

$$\mathbf{P}(t) = \sum A_i(t)\mathbf{P}_i,\ t_{\min} \leq t < t_{\max}$$

Give the conditions on the functions A_i that are needed to ensure that:

(*i*) Translation of all of the points by some vector, $\mathbf{P}'_i = \mathbf{P}_i + \Delta\mathbf{P}$, causes a translation of the curve by the same vector, $\mathbf{P}'(t) = \mathbf{P}(t) + \Delta\mathbf{P}$. [2 marks]

(*ii*) The curve lies within the convex hull of the control points. [2 marks]

(*iii*) The curve passes through one of the control points, $\mathbf{P}_j$. [2 marks]

(*b*) The knot vector $[0, 0, 0, 1, 1, 1]$ defines a quadratic B-spline with three control points. Derive the equations of and graph the three basis functions from this knot vector. [6 marks]

(*c*) The basis functions derived in part (*b*) can be used, in a NURBS curve, to reproduce exactly a quarter-circle. Recall that a NURBS curve can be written as:

$$\mathbf{P}(t) = \frac{\sum_{i=1}^{n+1} N_{i,k}(t)\mathbf{P}_i h_i}{\sum_{i=1}^{n+1} N_{i,k}(t) h_i}, t_{\min} \leq t < t_{\max}$$

where h_i is the *homogeneous co-ordinate* associated with point $\mathbf{P}_i$. Place the three control points at $\mathbf{P}_1 = (1, 0)$, $\mathbf{P}_2 = (1, 1)$, $\mathbf{P}_3 = (0, 1)$.

(*i*) Sketch the NURBS curve for the case $h_1 = h_2 = h_3 = 1$ [1 mark]

(*ii*) Calculate the magnitude of the maximum error between the curve in (*c*)(*i*) and a perfect circle of radius 1 centred at $(0, 0)$. [2 marks]

(*iii*) Sketch the NURBS curve for the case $h_1 = h_3 = 1$, $h_2 = 0$. [1 mark]

(*iv*) Sketch the NURBS curve for the limit case $h_1 = h_3 = 1$, $h_2 \to \infty$. [1 mark]

(*v*) Derive the value for h_2 that makes the NURBS curve perfectly match a quarter circle of radius 1 centred at $(0, 0)$. [3 marks]