

COMPUTER SCIENCE TRIPOS Part IB – 2013 – Paper 6

7 Mathematical Methods for Computer Science (JGD)

- (a) For vectors $u, v \in V$ in linear space $V = \mathbb{R}^n$ with $u = (u_1, u_2, \dots, u_n)$, define the Euclidean norm $\|u\|$, and state the triangle inequality for $\|u + v\|$. [2 marks]
- (b) Define cyclical convolution of two periodic sequences $f[n]$ and $g[n]$. [2 marks]
- (c) If $\Psi(x)$ is a generating (or “mother”) wavelet, give the dyadic shifting and scaling operations that generate her “daughter” wavelets $\Psi_{jk}(x)$ in terms of dilates j and translates k of $\Psi(x)$. [2 marks]
- (d) Why is the dyadic property of wavelets useful for analysing naturally-arising data that often exhibits self-similarity across scales? [2 marks]
- (e) Derive the Fourier series of a periodic triangle wave, $f(x) = |x|$ for $x \in [-\pi, \pi]$ [4 marks]
- (f) The Modulation Theorem asserts that if $f(x)$ has Fourier transform $F(\omega)$, then modulating $f(x)$ at frequency c (multiplying it by e^{icx}) simply shifts its transform up by c to become $F(\omega - c)$. Prove this, and explain one important practical application of this property. [4 marks]
- (g) Show how Fourier methods enable solution of differential equations such as the following, in which the function $g(x)$ is known (hence its Fourier transform $G(\omega)$ can be computed), and a, b, c are constant coefficients. Derive an expression for $f(x)$ that solves this differential equation.

$$a \frac{d^2 f(x)}{dx^2} + b \frac{df(x)}{dx} + cf(x) = g(x)$$

[4 marks]