

8 Prolog (ACR)

- (a) The propositional logic formula $A \wedge B$ can be represented by the Prolog term `and(lit(A),lit(B))`.

Describe a scheme based on this example for representing an arbitrary propositional logic formula in Prolog. Demonstrate your scheme by encoding the formula $\neg(\neg P \wedge (Q \vee \neg(R \wedge S)))$. [4 marks]

- (b) A formula is in Conjunctive Normal Form (CNF) if it is expressed as a conjunction (\wedge -ing) of clauses, where each clause is a disjunction (\vee -ing) of literals.

Write a Prolog program for converting a propositional logic formula into CNF by implementing the following procedure:

- (i) Push negations inwards until each applies only to a literal using *De Morgan's laws*: $\neg(A \vee B) \simeq \neg A \wedge \neg B$ and $\neg(A \wedge B) \simeq \neg A \vee \neg B$ [5 marks]
- (ii) Remove double negations of literals: $\neg\neg A \simeq A$ [1 mark]
- (iii) Distribute one disjunction from the formula over a conjunction or fail if no such disjunction exists: $A \vee (B \wedge C) \simeq (A \vee B) \wedge (A \vee C)$ [6 marks]
- (iv) Repeatedly apply the distribution step until no more distribution can be done [4 marks]

Ensure that your predicates behave appropriately with backtracking, avoid over-use of cut, and are commented appropriately. Minor syntactic errors will not be penalised.