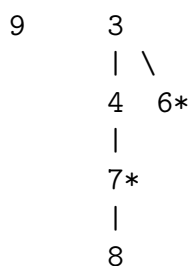


2 Algorithms II (FMS)

(a) Draw a clear sequence of commented snapshots showing what happens when applying the following sequence of operations to the two-tree Fibonacci heap pictured below, where asterisks denote marked nodes.

- insert key 5
- extract the minimum
- decrease key 8 by 2



[You should draw one snapshot after each operation; and also, if it makes your explanation clearer, between 0 and 3 snapshots at key points during each operation.]

[7 marks]

(b) Explain how “cascading cuts” work.

[2 marks]

(c) Explain in clear detail why “cascading cuts” are necessary to achieve the Fibonacci heap’s performance. What methods would be slower without cascading cuts, and by how much?

[5 marks]

(d) “If, in a Fibonacci heap, we never call `decreaseKey()` or `delete()`, then at any time the degree (number of children) of any node is at most $\lceil \lg n \rceil$, where n is the total number of nodes in the heap at that time.” True or false? Give a proof or a counterexample.

[6 marks]