COMPUTER SCIENCE TRIPOS Part IB – 2013 – Paper 3

1 Algorithms II (FMS)

The context for this question is the search for a minimum spanning tree (MST) for a weighted connected graph.

(*a*) Give a clear definition of the following MST technical expressions, describing also the type of X and Y: "X respects Y", "z is a safe edge". [3 marks]

(*b*) For each of the following statements, say whether it is true or false and then support your argument with a correctness proof or a small counterexample as appropriate. [*You should give a specific graph, preferably small, if you are offering a counterexample.*]

(*i*) In a graph where all edge weights are positive, if a subset of edges connects all vertices and has minimum total weight, then it is a tree. [2 marks]

(*ii*) In a graph where edge weights may be positive or negative, if a subset of edges connects all vertices and has minimum total weight, then it is a tree. [5 marks]

(*iii*) Let T be a minimum spanning tree, C be a cut and e be the lightest edge crossing the cut. Assume $e \notin T$. Call f one of the edges of T that crosses the cut (one must exist because T spans all vertices). Then the set of edges $T \cup \{e\} \setminus \{f\}$ is also a tree. [10 marks]