7 Probability (RJG)

Suppose that $N$ is a random variable taking values $j = 0, 1, 2 \ldots$ with

$$P(N = j) = (1 - \rho)\rho^j$$

and where $0 < \rho < 1$.

(a) Show that $P(N = j)$ is a probability mass function. [2 marks]

(b) For $k = 0, 1, 2, \ldots$ derive an expression for $P(N > k)$. [2 marks]

(c) Derive the probability generating function, $G_N(z)$, for the random variable $N$, stating carefully any conditions required for it to be well-defined and use it to determine

(i) $\mathbb{E}(N)$

(ii) Var($N$) [8 marks]

(d) For a random variable $X$, which takes non-negative integer values, show that

$$\mathbb{E}(X) = \sum_{k=0}^{\infty} P(X > k).$$

[6 marks]

(e) By evaluating $\sum_{k=0}^{\infty} P(N > k)$ show that this expression equals the value derived in part (c)(i) for $\mathbb{E}(N)$. [2 marks]