3 Discrete Mathematics I (SS)

(*a*) Consider the following assertions about the sets A, B and C. Write them down in the language of predicate logic. Use only the constructions of predicate logic ($\forall$, $\exists$, $\neg$, $\Rightarrow$, $\wedge$, $\vee$) and the element-of symbol ($\in$). Do *not* use derived notions ($\cap$, $\cup$, $=$, etc.).

Example: "A is a subset of B" can be formalized as $\forall x. x \in A \implies x \in B$.

(*i*) The sets A and B are equal.

(*ii*) Every element of A is in the set B or the set C.

(*iii*) If A is disjoint from B then B and C overlap.

[6 marks]

(*b*) State the principle of induction over lists. Use the language of predicate logic.

[2 marks]

(*c*) Consider the following functions over lists of integers, written in ML syntax.

```
fun app([],ys) = ys
  | app(x::xs,ys) = x::app(xs,ys);

fun rev([]) = []
  | rev(x::xs) = app(rev(xs),x::[]);

fun revapp([],ys) = ys
  | revapp(x::xs,ys) = revapp(xs,x::ys);
```

Prove that

$$\forall \texttt{xs}.\ \texttt{revapp(xs,[])} = \texttt{rev(xs)}$$

Your proof should be clear but it does not need to be a structured proof. You may use the abbreviation `xs @ ys` for `app(xs,ys)`. You may assume the following facts.

$$\forall \texttt{xs}.\ \texttt{xs @ []} = \texttt{xs} \qquad \forall \texttt{xs}, \texttt{ys}, \texttt{zs}.\ \texttt{xs @ (ys @ zs)} = \texttt{(xs @ ys) @ zs}$$

Hint: first use induction to show that

$$\forall \texttt{xs}.\ \forall \texttt{ys}.\ \texttt{revapp(xs,ys)} = \texttt{app(rev(xs),ys)}.$$

[12 marks]