5 Computer Vision (JGD)

(a) The following very useful operator is often applied to an image \( I(x, y) \) in computer vision algorithms, to generate a related “image” \( g(x, y) \) for analysis:

\[
g(x, y) = \int_{\alpha}^{\beta} \int_{\beta}^{\alpha} \nabla^2 e^{-((x-\alpha)^2+(y-\beta)^2)/\sigma^2} I(\alpha, \beta) \, d\alpha \, d\beta
\]

where

\[
\nabla^2 = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)
\]

(i) Give the general name for this type of mathematical operation, and the chief purpose that it serves in computer vision. [1 mark]

(ii) What structures are expected at places \((x, y)\) in the image \( I(x, y) \) where the operator output \( g(x, y) \) undergoes zero-crossings? [1 mark]

(iii) What is the significance of the parameter \( \sigma \)? If you increased its value, would there be more or fewer points \((x, y)\) where \( g(x, y) = 0? \) [2 marks]

(iv) Describe the effect of the above operator in terms of the two-dimensional Fourier domain. What is the Fourier terminology for this image-domain operator? What are its general effects as a function of frequency, and as a function of orientation? [2 marks]

(v) If the computation of \( g(x, y) \) were to be implemented entirely by Fourier methods, would its complexity be greater or less than the image-domain operation, and why? What would be the trade-offs involved? [2 marks]

(vi) If the image \( I(x, y) \) has 2D Fourier Transform \( F(u, v) \), provide an expression for \( G(u, v) \), the 2D Fourier Transform of the operator output \( g(x, y) \) in terms of \( F(u, v) \), the Fourier plane variables \( u, v \), some constants, and the parameter \( \sigma \). [2 marks]

(b) Briefly define each of the following concepts as it relates to vision:

(i) “signal-to-symbol converter” [2 marks]

(ii) “inverse graphics” [2 marks]

(iii) quadrature demodulator [2 marks]

(iv) volumetric coordinates [2 marks]

(v) correspondence problem [2 marks]