

COMPUTER SCIENCE TRIPOS Part IB – 2012 – Paper 6

8 Mathematical Methods for Computer Science (RJG)

- (a) Consider the Markov Chain,  $X_n$ , on the states  $i = 0, 1, 2, \dots$  with transition matrix given by

$$\begin{aligned}p_{i,i-1} &= p & i &= 1, 2, \dots \\p_{i,i+1} &= 1 - p & i &= 0, 1, \dots \\p_{0,0} &= p\end{aligned}$$

where  $0 < p < 1$ .

- (i) Show that the Markov chain is irreducible. [2 marks]
- (ii) Show that the Markov chain is aperiodic. [2 marks]
- (iii) Find a condition on  $p$  to make the Markov chain positive recurrent and find the stationary distribution in this case. [6 marks]
- (b) Consider the PageRank algorithm.
- (i) Describe PageRank as a Markov chain model for the motion between nodes in a graph where the nodes correspond with web pages. [5 marks]
- (ii) Explain the main mathematical results that underpin the relevance of PageRank to a notion of web page *importance*. [5 marks]