

COMPUTER SCIENCE TRIPOS Part IA – 2012 – Paper 2

6 Discrete Mathematics II (GW)

(a) State the principle of rule induction. [2 marks]

(b) Let X be the smallest subset of $\mathbb{N}_0 = \{0, 1, 2, 3, \dots\}$ such that

$2 \in X$ and $6 \in X$, and

if $x \in X$ and $y \in X$ then their product $x \times y \in X$.

(i) Using rule induction show $X \subseteq \{2^m 3^n \mid m, n \in \mathbb{N}_0\}$. [4 marks]

(ii) Is $18 \in X$? Justify your claim. [4 marks]

(iii) Describe a property $Q(m, n)$, where $m, n \in \mathbb{N}_0$, such that

$$X = \{2^m 3^n \mid m, n \in \mathbb{N}_0 \ \& \ Q(m, n)\}$$

[5 marks]

(iv) Prove your claim for part (b)(iii). [5 marks]