4 Discrete Mathematics I (SS)

(a) Let $A$ be the set $\{1, 2, 3\}$. The following relations are subsets of $A \times A$. Draw them as directed graphs.

(i) $R_1 = \{(x, y) \mid x \in A \land y \in A \land x - y = 1\}$

(ii) $R_2 = \{(x, y) \mid x \in A \land y \in A \land x - y \geq 1\}$

(iii) $R_3 = \{(x, y) \mid x \in A \land y \in A \land x - y = 0\}$

(iv) $R_4 = \{(x, y) \mid x \in A \land y \in A \land -(x - y = 0)\}$

(v) $R_5 = \{(x, y) \mid x \in A \land y \in A \land \forall u. \exists v. x + u = y + v\}$
   where $u$ and $v$ range over the integers

(vi) $R_6 = \{(x, y) \mid x \in A \land y \in A \land \exists u. \forall v. x + u = y + v\}$
   where $u$ and $v$ range over the integers [6 marks]

(b) Write down what it means for a relation to be transitive. Which of the relations in part (a) are transitive? [3 marks]

(c) Write down the introduction and elimination rules for the universal quantifier in structured proof. [3 marks]

(d) Recall the following introduction and elimination rules for implication.

\[
\begin{array}{l}
m. \text{Assume } P \\
\quad \ldots \\
\quad \ldots \\
n. \text{Q from } \ldots \text{ by } \ldots \\
n + 1. \text{P } \Rightarrow \text{Q from } m-n, \\
\quad \text{by } \Rightarrow \text{-introduction.}
\end{array}
\quad \begin{array}{l}
l. \text{P } \Rightarrow \text{Q from } \ldots \text{ by } \ldots \\
\quad \ldots \\
\quad \ldots \\
m. \text{P from } \ldots \text{ by } \ldots \\
\quad \ldots \\
n. \text{Q from } l, m \\
\quad \text{by } \Rightarrow \text{-elimination.}
\end{array}
\]

Write down a structured proof of the following statement.

\[(\forall a. P(a) \Rightarrow Q(a)) \Rightarrow ((\forall b. Q(b) \Rightarrow R(b)) \Rightarrow (\forall c. P(c) \Rightarrow R(c)))\]

[8 marks]