3 Discrete Mathematics I (SS)

(a) Which of the following formulas are tautologies? Explain what is meant by “tautology” and write down truth tables to justify your answers.

(i) \( p \Rightarrow q \)

(ii) \( (p \Rightarrow q) \Rightarrow p \)

(iii) \( ((p \Rightarrow q) \Rightarrow p) \Rightarrow p \) \[4 \text{ marks}\]

(b) Recall the following introduction and elimination rules for implication.

\[
\begin{align*}
\ldots \\
m. & \text{ Assume } P \\
\ldots \\
n. & \text{ Q from } \ldots \text{ by } \ldots \\
n + 1. & \text{ P } \Rightarrow \text{ Q from } m-n, \\
& \text{ by } \Rightarrow\text{-introduction.}
\end{align*}
\]

\[
\begin{align*}
\ldots \\
l. & \text{ P } \Rightarrow \text{ Q from } \ldots \text{ by } \ldots \\
\ldots \\
m. & \text{ P from } \ldots \text{ by } \ldots \\
\ldots \\
n. & \text{ Q from } l, m \\
& \text{ by } \Rightarrow\text{-elimination.}
\end{align*}
\]

(i) Write down the elimination rules for negation and falsity. \[3 \text{ marks}\]

(ii) Using the four rules above, write down a structured proof of

\( \neg p \Rightarrow (p \Rightarrow q) \) \[4 \text{ marks}\]

(iii) Write down the principle of proof by contradiction. \[2 \text{ marks}\]

(iv) Using everything from part (b) so far, write down a structured proof of

\( ((p \Rightarrow q) \Rightarrow p) \Rightarrow p \) \[7 \text{ marks}\]

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