2011 Paper 9 Question 1

Computer Systems Modelling

(*a*) Suppose that the random variable X has an exponential distribution with parameter $\lambda > 0$.

(*i*) What are the probability density function, $f_X(x)$, and the probability distribution function, $F_X(x)$, for the random variable X? [2 marks]

(*ii*) Derive the mean and variance of the random variable X and determine its coefficient of variation. [3 marks]

(*iii*) Show that the random variable X obeys the *memoryless property*

$$P(X > t + s | X > t) = P(X > s)$$

for all $s, t > 0$. [2 marks]

(*iv*) Use the inverse transform method to derive a method to simulate random variables, X_i, indexed by $i = 1, 2, \ldots$ from an exponential distribution with parameter $\lambda > 0$ given a sequence of pseudo-random values U_i from the uniform distribution $U(0, 1)$. [3 marks]

(*b*) Suppose that $X_1, X_2, \ldots$ is a sequence of independent and identically distributed random variables with each random variable X_i having a marginal distribution that is an exponential distribution with parameter $\lambda > 0$. Let $S_n = \sum_{i=1}^{n} X_i$ where n is a positive integer and let the random variable $N(t)$ for $t > 0$ be the number of events in a Poisson Process with parameter $\lambda > 0$ that occur in the time interval $(0, t)$.

(*i*) State the probability distribution of $N(t)$. [2 marks]

(*ii*) State a relation between S_n and $N(t)$. [2 marks]

(*iii*) Derive the probability density of the random variable S_n. [6 marks]