Hoare Logic

The programming language $L$ consists of commands $C$ composed from assignments $V := E$ (where $E$ is an expression) using sequences $C_1; C_2$, conditionals $\text{IF } S \text{ THEN } C_1 \text{ ELSE } C_2$ (where $S$ is statement) and while-loops $\text{WHILE } S \text{ DO } C$.

(a) Devise a command $\text{SKIP}$ in $L$ that has no effect and, for arbitrary $P$, prove using the Hoare logic axioms and rules for the constructs of $L$ that $\vdash \{P\} \text{SKIP} \{P\}$. [4 marks]

(b) Devise a one-armed conditional $\text{IF } S \text{ THEN } C$ built only from $S$, $C$ and constructs of $L$ and show using the Hoare logic for $L$ that if $\vdash \{P \land S\} C \{Q\}$ and $\vdash P \land \neg S \Rightarrow Q$ then $\vdash \{P\} \text{IF } S \text{ THEN } C \{Q\}$. [6 marks]

(c) Define a command $\text{MAGIC}$ in $L$ that has the property $\vdash \{P\} \text{MAGIC} \{Q\}$ for any precondition $P$ and postcondition $Q$. Prove that your definition of $\text{MAGIC}$ has this property using the Hoare logic for $L$. [10 marks]