

2011 Paper 8 Question 12

Computer Systems Modelling

(a) Consider an open Jackson queueing network.

(i) Give a description of an open Jackson network. Explain the parameters that specify the network and the state space that you would use to model its behaviour. [2 marks]

(ii) Derive the traffic equations for the arrival rates λ_i at each node i in the network. [2 marks]

(iii) What is the condition for the existence of an equilibrium distribution? [2 marks]

(iv) State Jackson's Theorem for an open Jackson network. [2 marks]

(b) Now consider the $M/M/m/m$ loss system with traffic intensity ρ .

(i) Show that the steady state loss probability, $E(\rho, m)$, that all servers are occupied is given by

$$E(\rho, m) = \frac{\rho^m/m!}{\sum_{i=0}^m \rho^i/i!}$$

[6 marks]

(ii) Show that $E(\rho, m)$ solves the recurrence relation

$$E(\rho, m) = \frac{\rho E(\rho, m-1)}{m + \rho E(\rho, m-1)}$$

with the boundary condition $E(\rho, 0) = 1$ and comment on why the recurrence relation is useful in practice. [6 marks]