# 2011 Paper 6 Question 8

## Mathematical Methods for Computer Science

Let $f[n]$ be a periodic sequence of period $N$ with $N$-point Discrete Fourier Transform (DFT) $F[k]$ given by

$$F[k] = \sum_{n=0}^{N-1} f[n]e^{-2\pi ink/N}$$

and inverse transform given by

$$f[n] = \frac{1}{N}\sum_{k=0}^{N-1} F[k]e^{2\pi ink/N}$$

Define the *power*, $P$, of a periodic sequence, $f[n]$, of period $N$ by

$$P = \frac{1}{N}\sum_{n=0}^{N-1} |f[n]|^2$$

(*a*) For each fixed $k$ show that the periodic sequence $\frac{1}{N}F[k]e^{2\pi ink/N}$ has power $|F[k]|^2/N^2$. [5 marks]

(*b*) If $g[n]$ is a periodic sequence of period $N$ with $N$-point DFT $G[k]$ show that

$$\sum_{n=0}^{N-1} f[n]\overline{g[n]} = \frac{1}{N}\sum_{k=0}^{N-1} F[k]\overline{G[k]}$$

[5 marks]

(*c*) Show that the power of the periodic sequence $f[n]$ is equal to $\frac{1}{N^2}\sum_{k=0}^{N-1} |F[k]|^2$.

[5 marks]

(*d*) Suppose that $f[n]$ is the periodic sequence given by $f[n] = \sin(2\pi n/N)$ of period $N$. Recall that $\sin(\theta) = (e^{i\theta} - e^{-i\theta})/2i$.

(*i*) Find $F[k]$ the $N$-point DFT of $f[n]$. [3 marks]

(*ii*) Show that the power of $f[n]$ is 1/2. [2 marks]