2011 Paper 6 Question 5

Logic and Proof

(*a*) Recently, automated theorem provers based on the saturation algorithm have become very powerful tools.

(*i*) Exhibit a proof by resolution of the following formula in first-order logic. Include the conversion into a set of clauses and provide brief justification for each step of the proof.

$$\forall x(P(x) \to Q(x)) \to (\exists y P(y) \to \exists z Q(z))$$

[6 marks]

(*ii*) Prove $P(s(s(s(0))))$ by *linear* resolution from the following assumptions:

$$\forall x((P(x) \land Q(x)) \to P(s(x)))$$

$$\forall x(P(x) \to Q(x))$$

$$P(0)$$

[7 marks]

(*b*) Binary decision diagrams (BDDs) can be used to represent formulae in propositional logic.

Show the steps in the recursive construction of a BDD, ordered alphabetically, for the following formula:

$$((P \land Q) \lor R) \to (Q \lor R)$$

[7 marks]