2011 Paper 6 Question 4

Computation Theory

(*a*) State precisely what it means for a function $f : \mathbb{N}^k \to \mathbb{N}$ to be *primitive recursive*, giving exact definitions for all operations you use. [5 marks]

(*b*) State precisely what it means for a function $f : \mathbb{N}^k \to \mathbb{N}$ to be λ-*definable*. [5 marks]

(*c*) For each of the following functions, show (using the definitions you gave) that it is primitive recursive and λ-definable.

(*i*) The function $square : \mathbb{N} \to \mathbb{N}$ given by $square(x) = x^2$. [4 marks]

(*ii*) The function $fact : \mathbb{N} \to \mathbb{N}$ given by $fact(x) = x!$. [4 marks]

(*d*) Give a definition of a function that is λ-definable but not primitive recursive. [2 marks]