(a) Explain how we can associate a unique number $\lceil P \rceil$ to each register machine program $P$. [5 marks]

(b) Consider the following two partial functions $S : \mathbb{N} \rightarrow \mathbb{N}$ and $T : \mathbb{N} \rightarrow \mathbb{N}$ on the natural numbers.

$$S([P]) = \begin{cases} n & \text{if the program } P \text{ when started with 0 in all registers} \\ & \text{halts after } n \text{ steps} \\ 0 & \text{otherwise.} \end{cases}$$

$$T([P]) = \begin{cases} n & \text{if the program } P \text{ when started with 0} \\ & \text{in all registers halts after } n \text{ steps} \\ undefined & \text{otherwise.} \end{cases}$$

(i) Which, if any, of $S$ and $T$ is computable and which is uncomputable? [4 marks]

(ii) Give full justification for your answers above. State carefully any standard results that you use. [11 marks]