2011 Paper 6 Question 3

Computation Theory

(*a*) Explain how we can associate a unique number $\lceil P \rceil$ to each register machine program P. [5 marks]

(*b*) Consider the following two partial functions $S : \mathbb{N} \rightharpoonup \mathbb{N}$ and $T : \mathbb{N} \rightharpoonup \mathbb{N}$ on the natural numbers.

$$S(\lceil P \rceil) = \begin{cases} n & \text{if the program } P \text{ when started with 0 in all registers} \\ & \text{halts after } n \text{ steps;} \\ 0 & \text{otherwise.} \end{cases}$$

$$T(\lceil P \rceil) = \begin{cases} n & \text{if the program } P \text{ when started with 0} \\ & \text{in all registers halts after } n \text{ steps;} \\ \textit{undefined} & \text{otherwise.} \end{cases}$$

(*i*) Which, if any, of S and T is computable and which is uncomputable? [4 marks]

(*ii*) Give full justification for your answers above. State carefully any standard results that you use. [11 marks]