Digital Electronics

(a) Simplify the following expressions using Boolean algebra:

(i) \[ F = A \cdot B \cdot \overline{C} + \overline{A} \cdot B \cdot \overline{C} + A \cdot B \cdot C + A \cdot B \cdot \overline{C} \]

(ii) \[ F = (X + Y) \cdot (\overline{X} + Y + Z) \cdot (X + Y + Z) \]

(iii) \[ F = (A \cdot D + \overline{A} \cdot C) \cdot \overline{B} \cdot (C + B \cdot \overline{D}) \]

[6 marks]

(b) Give the truth table for an encoder that accepts a sign bit, \( S \), and two magnitude bits \( X_0, X_1 \) and gives a three-bit output \( Y_2, Y_1, Y_0 \) that are the two’s complement encoding of the input. [4 marks]

(c) Using a Karnaugh map, simplify the following expression to yield a solution in a sum-of-products form:

\[ Y = \overline{A} \cdot B \cdot \overline{C} \cdot D + A \cdot \overline{B} \cdot \overline{C} \cdot D + A \cdot \overline{D} + \overline{A} \cdot B \cdot \overline{D} \]

What problem may exist with a practical realisation of this solution, and how may it be cured? [5 marks]

(d) Simplify the following expression using a Karnaugh map to yield a solution in product-of-sums form and implement it using only NOR gates assuming complemented input variables are available:

\[ Y = (B + \overline{C} + \overline{D}) \cdot (\overline{A} + B + \overline{C}) \cdot (A + B + \overline{D}) \cdot (A + \overline{B} + \overline{C}) \]

Neglect any potential problems in the practical realisation of your solution. [5 marks]