COMPUTER SCIENCE TRIPOS Part IA

Tuesday 7 June 2011 1.30 to 4.30

COMPUTER SCIENCE Paper 2

*Answer **one** question from each of Sections A, B and C, and **two** questions from Section D.*

*Submit the answers in five **separate** bundles, each with its own cover sheet. On each cover sheet, write the numbers of **all** attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script paper
Blue cover sheets
Tags

SPECIAL REQUIREMENTS
Approved calculator permitted

SECTION A

1 Digital Electronics

(*a*) Simplify the following expressions using Boolean algebra:

(*i*) $F = A\,.\,\overline{B}\,.\,\overline{C} + \overline{A}\,.\,B\,.\,\overline{C} + \overline{A}\,.\,\overline{B}\,.\,C + A\,.\,B\,.\,C$

(*ii*) $F = (X + Y)\,.\,(\overline{X} + Y + Z)\,.\,(\overline{X} + Y + \overline{Z})$

(*iii*) $F = (A\,.\,D + \overline{A}\,.\,C)\,.\,[\overline{B}\,.\,(C + B\,.\,\overline{D})]$

[6 marks]

(*b*) Give the truth table for an encoder that accepts a sign bit, S, and two magnitude bits X_0, X_1 and gives a three-bit output Y_2, Y_1, Y_0 that are the two's complement encoding of the input. [4 marks]

(*c*) Using a Karnaugh map, simplify the following expression to yield a solution in a sum-of-products form:

$$Y = \overline{A}\,.\,\overline{B}\,.\,\overline{C}\,.\,D + A\,.\,\overline{B}\,.\,\overline{C}\,.\,D + A\,.\,\overline{D} + \overline{A}\,.\,B\,.\,\overline{D}$$

What problem may exist with a practical realisation of this solution, and how may it be cured? [5 marks]

(*d*) Simplify the following expression using a Karnaugh map to yield a solution in product-of-sums form and implement it using only NOR gates assuming complemented input variables are available:

$$Y = (B + \overline{C} + \overline{D})\,.\,(\overline{A} + B + \overline{C})\,.\,(A + B + \overline{D})\,.\,(A + \overline{B} + \overline{C})$$

Neglect any potential problems in the practical realisation of your solution.

[5 marks]

2 Digital Electronics

(*a*) Show how two 2-input NOR gates can be connected together to implement an RS latch. Describe its operation and give its truth table. [6 marks]

(*b*) Draw the state diagram for a synchronous modulo-4 up/down counter. The counter has two control inputs: M is set at logic "0" to cause the counter to count up, and at logic "1" to cause the counter to count down; E is set at logic "1" to enable the counter to count and at logic "0" to cause the counter to hold its current state. [4 marks]

(*c*) A synchronous binary up-counter having the state sequence

$$1, 2, 3, 4, 5, 6, 1, 2, \ldots$$

is to be implemented using three D-type flip-flops. The flip-flop outputs are designated Q_2, Q_1 and Q_0, where Q_0 represents the least significant digit of the count.

(*i*) Give simplified expressions for the required next-state logic, making use of any unused states. Does this counter self-start? [6 marks]

(*ii*) Give the new simplified expression required for D_0 (the D-input of flip-flop Q_0) if the counter is now required to return to a count of 1 if an unused state is entered. [4 marks]

SECTION B

3 Operating Systems

(*a*) In the context of the protection of computer systems:

(*i*) What is meant by *access control*? [1 mark]

(*ii*) What is an *access control list*? [2 marks]

(*iii*) What is a *capability*? [2 marks]

(*iv*) How is access control managed in the UNIX file system? [5 marks]

(*v*) How is access control managed in Windows NT? [5 marks]

(*b*) Describe how you can use *page protection* bits to implement a not-recently-used page replacement scheme. [5 marks]

4 Operating Systems

(*a*) In the context of memory management:

(*i*) What is the *address binding* problem? [1 mark]

(*ii*) The address binding problem can be solved at compile time, load time or run time. For **each** case, explain what form the solution takes, and give one advantage and one disadvantage. [3 marks each]

(*iii*) Under which circumstances do *external* and *internal* fragmentation occur? How can each be handled? [4 marks]

(*iv*) What is the purpose of a *translation lookaside buffer* (TLB)? [2 marks]

(*b*) Describe how UNIX handles user authentication. [4 marks]

SECTION C

5 Discrete Mathematics II

Let A and B be sets. Let $F \subseteq \mathcal{P}(A) \times B$. So a typical element of F is a pair (X, b) where $X \subseteq A$ and $b \in B$. Define the function

$$f : \mathcal{P}(A) \to \mathcal{P}(B)$$

to be such that

$$f(x) = \{b \mid \exists X \subseteq x.\ (X, b) \in F\}$$

for $x \in \mathcal{P}(A)$.

(*a*) Show

$$\text{if } x \subseteq y \text{ then } f(x) \subseteq f(y)$$

for all $x, y \in \mathcal{P}(A)$. [3 marks]

(*b*) Suppose

$$x_0 \subseteq x_1 \subseteq \cdots \subseteq x_n \subseteq \cdots$$

is a chain of subsets in $\mathcal{P}(A)$. Recall $\bigcup_{n \in \mathbb{N}_0} x_n = \{a \mid \exists n \in \mathbb{N}_0.\ a \in x_n\}$. Show that

$$\bigcup_{n \in \mathbb{N}_0} f(x_n) \subseteq f(\bigcup_{n \in \mathbb{N}_0} x_n)$$

[Hint: Use part (*a*).] [4 marks]

(*c*) Assume now that $F \subseteq \mathcal{P}_{\text{fin}}(A) \times B$ where $\mathcal{P}_{\text{fin}}(A)$ consists of the finite subsets of A. So now a typical element of F is a pair (X, b) where X is a *finite* subset of A and $b \in B$. Suppose $x_0 \subseteq x_1 \subseteq \cdots \subseteq x_n \subseteq \cdots$ is a chain of subsets in $\mathcal{P}(A)$. Show that

$$f(\bigcup_{n \in \mathbb{N}_0} x_n) \subseteq \bigcup_{n \in \mathbb{N}_0} f(x_n)$$

Deduce

$$f(\bigcup_{n \in \mathbb{N}_0} x_n) = \bigcup_{n \in \mathbb{N}_0} f(x_n) \qquad (\dagger)$$

[6 marks]

(*d*) Show that $(\dagger)$ need not hold if the set X in elements (X, b) of F is infinite.
[7 marks]

6 Discrete Mathematics II

Let E be a set. Assume $\mathcal{F} \subseteq \mathcal{P}(E)$ satisfies the two conditions

1. $\forall X \subseteq \mathcal{F}.\ \bigcup X \in \mathcal{F}$
2. $\forall X \subseteq \mathcal{F}.\ \bigcap X \in \mathcal{F}$

Recall

$$\bigcup X =_{\mathrm{def}} \{e \in E \mid \exists x \in X.\ e \in x\} \quad \text{and} \quad \bigcap X =_{\mathrm{def}} \{e \in E \mid \forall x \in X.\ e \in x\}$$

(*a*) Explain why $\emptyset \in \mathcal{F}$ and $E \in \mathcal{F}$. [2 marks]

(*b*) Define the binary relation $\lesssim$ on E by

$$e' \lesssim e \ \text{ iff } \ \forall x \in \mathcal{F}.\ e \in x \ \Rightarrow e' \in x$$

for $e, e' \in E$. State clearly what it would mean for $\lesssim$ to be reflexive and transitive. Show $\lesssim$ is reflexive and transitive. [5 marks]

(*c*) For $e \in E$, define

$$[e] = \bigcap \{x \in \mathcal{F} \mid e \in x\}$$

Explain why $[e] \in \mathcal{F}$. Show

$$[e] = \{e' \mid e' \lesssim e\}$$

[6 marks]

(*d*) Say a subset z of E is *down-closed* iff

$$e' \lesssim e \ \&\ e \in z \quad \Rightarrow \quad e' \in z$$

for all $e, e' \in E$. Show $\mathcal{F}$ consists of precisely the down-closed subsets of E by showing:

(*i*) any $x \in \mathcal{F}$ is down-closed; [3 marks]

(*ii*) for any down-closed subset z of E,

$$z = \bigcup \{[e] \mid e \in z\}$$

and hence $z \in \mathcal{F}$ (why?). [4 marks]

SECTION D

7 Probability

(*a*) State the *probability mass function* for a Poisson random variable with parameter $\lambda > 0$. [2 marks]

(*b*) Define the *probability generating function*, $G_X(z)$, of a random variable X taking values in $\{0, 1, 2, \ldots\}$ and derive an expression for $G_X(z)$ in the case where $X \sim \text{Pois}(\lambda)$ with $\lambda > 0$. [4 marks]

(*c*) Show the following result

$$G_X^{(r)}(1) = E(X(X-1)\cdots(X-r+1))$$

where r is a positive integer and $G_X^{(r)}(1)$ denotes the rth derivative of $G_X(z)$ with respect to z evaluated at $z = 1$. [4 marks]

(*d*) Using the result in part (*c*) derive the mean and variance of a Poisson random variable with parameter $\lambda > 0$. [4 marks]

(*e*) Show the result that if X and Y are two independent random variables with probability generating functions $G_X(z)$ and $G_Y(z)$, respectively, then

$$G_{X+Y}(z) = G_X(z)G_Y(z)$$

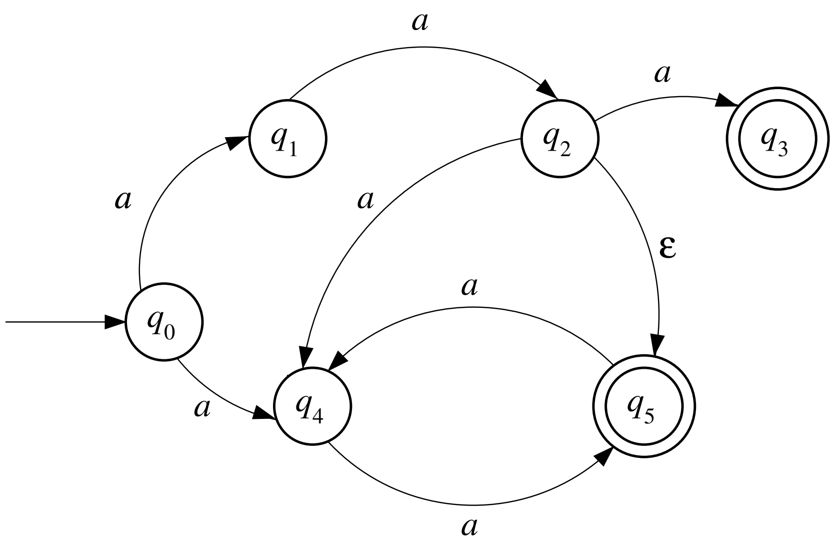
where $G_{X+Y}(z)$ is the probability generating function of $X + Y$. [2 marks]

(*f*) Show that if $\lambda_1, \lambda_2 > 0$ and $X \sim \text{Pois}(\lambda_1)$ and $Y \sim \text{Pois}(\lambda_2)$ are independent random variables then $X+Y \sim \text{Pois}(\lambda_1+\lambda_2)$. What are the mean and variance of $X + Y$? [4 marks]

8 Regular Languages and Finite Automata

(*a*) Give a regular expression r over the alphabet $\Sigma = \{a, b, c\}$ such that the language determined by r consists of all strings that contain at least one occurrence of each symbol in Σ. Briefly explain your answer. [5 marks]

(*b*) Let L be the language accepted by the following non-deterministic finite automaton with ε-transitions:



(*i*) Draw a deterministic finite automaton that accepts L.

(*ii*) Write down a regular expression that determines L.

Briefly explain your answers. [5 marks]

(*c*) Show that if a deterministic finite automaton M accepts any string at all, then it accepts one whose length is less than the number of states in M. [5 marks]

(*d*) Is the language

$$\{\, a^n b^\ell a^k \in \{a, b\}^* \mid k \geq n + \ell \,\}$$

regular? Justify your answer. [5 marks]

9 Software Design

Imagine that you are responsible for the design of a computer system that will be used to automate the definition, evaluation and examining of the academic content for a course in the Cambridge Computer Science Tripos. This system should allow the syllabus, lectures, supervision exercises and examination papers to be defined in consultation with a variety of stakeholders, including students and future employers.

(*a*) How would you go about determining the detailed requirements for this system? Be sure to mention any obstacles that you would expect to arise. [2 marks]

(*b*) Construct one or more UML use case diagrams *and* a single UML class diagram, showing the overall structure of a system that includes the elements described above. [10 marks]

(*c*) Using another type of UML diagram, illustrate the runtime behaviour of one of the use cases. [3 marks]

(*d*) Explain why you chose the specific UML diagram used in part (*c*). [1 mark]

(*e*) What precautions could you take to ensure that the introduction of the system was as smooth as possible? [2 marks]

(*f*) What technical precautions could you take to ensure that the system could be modified in response to future changes in regulations or user requirements? [2 marks]

END OF PAPER