

2010 Paper 8 Question 15

Types

- (a) Give brief explanations of the following concepts.
- (i) The relation *specialization* between Mini-ML type schemes and types. [2 marks]
 - (ii) A *principal type scheme* for a closed Mini-ML expression. [2 marks]
 - (iii) The *most general unifier* of two Mini-ML types. [2 marks]
- (b) State the Hindley–Damas–Milner theorem for the Mini-ML typeability problem. [3 marks]
- (c) What aspect of the Mini-ML type system facilitates proofs that expressions are *not* typeable? Illustrate your answer by explaining why $\lambda x((x x) x)$ is not typeable. [5 marks]
- (d) Consider adding “conjunction” types $\tau_1 \ \& \ \tau_2$ to Mini-ML with the following three typing rules:

$$\frac{\Gamma \vdash M : \tau_1 \quad \Gamma \vdash M : \tau_2}{\Gamma \vdash M : \tau_1 \ \& \ \tau_2}(\text{and}) \quad \frac{\Gamma \vdash M : \tau_1 \ \& \ \tau_2}{\Gamma \vdash M : \tau_1}(\text{pr}_1) \quad \frac{\Gamma \vdash M : \tau_1 \ \& \ \tau_2}{\Gamma \vdash M : \tau_2}(\text{pr}_2)$$

- (i) Show that $\lambda x((x x) x)$ is typeable in this extended system. [4 marks]
- (ii) Suggest an expression that is not typeable in the extended system. What, if anything, makes it difficult to prove that this is the case? [2 marks]