2010 Paper 8 Question 1

Advanced Graphics

(*a*) Discrete curvature

(*i*) Give each of the following: the *Gaussian curvature* at the exact centre of any face of a cube; the *Gaussian curvature* at any corner of a cube; and the *angle deficit* at any corner of a cube. [1 mark]

(*ii*) Sketch a picture of a closed manifold surface with total angle deficit -4π. The picture must be intelligible but you will not otherwise be marked on artistic skill. [2 marks]

(*iii*) If your hypothetical surface had 20 vertices and 20 faces then how many edges must it have? [2 marks]

(*b*) The convex hull

(*i*) In no more than ten sentences and/or half a page of pseudocode, describe a method for finding the convex hull of a set of n points in 3D. For full marks, give an algorithm that runs in $O(n^2)$ time or faster; partial marks will be given to any slower solution. You must give enough detail that a programmer with no knowledge of computational geometry could implement your algorithm. [4 marks]

(*ii*) Give the running time of your algorithm in big-O notation. [1 mark]

(*c*) Global illumination

(*i*) In no more than six sentences, describe **either** radiosity rendering **or** photon mapping. [3 marks]

(*ii*) In no more than six sentences, compare your chosen method with the other one. [2 marks]

(*iii*) Which of these two is an example of a *Monte Carlo* algorithm? [1 mark]

(*d*) Ray tracing

A perfectly reflective mirrored sphere, S, is centred at the origin $(0, 0, 0)$. Directly above it is a bright red $2 \times 2 \times 2$ cube, C, centred at $(0, 5, 0)$. The default background colour of the scene is blue. A ray-tracing ray R is fired from $(0, 1, 10)$ with direction $(0, 0, -1)$. The scene is lit by an ambient light source and there are no other objects in the scene. What is the maximum radius of S such that the colour calculated for R is red? Full marks for the correct answer; partial marks if you answer incorrectly but your work justifies your response. [4 marks]