Denotational Semantics

(a) For a domain $D$, recall that by Tarski’s Fixed-Point Theorem every continuous function $f \in (D \to D)$ has a least pre-fixed point $\text{fix}(f) \in D$.

Prove that the function $\text{fix} : (D \to D) \to D$ is continuous. \[10 \text{ marks}\]

(b) For a partially ordered set $(P, \sqsubseteq)$, let $(\text{Ch}(P), \sqsubseteq_{\text{ptw}})$ be the partially ordered set of chains in $P$ ordered pointwise. That is,

$$\text{Ch}(P) \overset{\text{def}}{=} \left\{ x = \{x_n\}_{n \in \mathbb{N}} \mid \text{ for all } i \leq j \text{ in } \mathbb{N}, x_i \sqsubseteq x_j \text{ in } P \right\}$$

and

$$x \sqsubseteq_{\text{ptw}} x' \overset{\text{def}}{\iff} x_n \sqsubseteq x'_n \text{ for all } n \in \mathbb{N}$$

Show that if $P$ is a domain then so is $\text{Ch}(P)$. \[10 \text{ marks}\]