2010 Paper 7 Question 2

Advanced Graphics

(a) For the knot vector $[0, 0, 0, 1, 1, 1]$:

(i) Derive all of the B-spline basis functions for the quadratic case ($k = 3$). [5 marks]

(ii) Place three control points, $P_1, \ldots, P_3$ in an equilateral triangle. Draw the B-spline curve, $P(t)$, defined by the control points and the basis functions derived from the knot vector. [2 marks]

(iii) Identify and label appropriately the locations on the curve in part (a)(ii) at which the parameter, t, is an integer. [1 mark]

(iv) Identify and express as a function of the control point locations the points at which the curve in part (a)(ii) touches the triangle defined by the control points, $P_1, \ldots, P_3$. [1 mark]

(b) For the uniform knot vector $[0, 1, 2, 3, 4, 5, 6]$:

(i) Sketch all of the quadratic ($k = 3$) B-spline basis functions. [2 marks]

(ii) Place four control points, $Q_1, \ldots, Q_4$ in a square. Draw the B-spline curve, $Q(t)$, defined by the control points and the basis functions derived from the knot vector. [2 marks]

(iii) Identify and label appropriately the locations on the curve in part (b)(ii) at which the parameter, t, is an integer. [1 mark]

(iv) Identify and express as a function of the control point locations the points at which the curve in part (b)(ii) touches the square defined by the control points, $Q_1, \ldots, Q_4$. [1 mark]

(v) Give the continuity of the curve at $t = 3$. [1 mark]

(c) Describe the corner-cutting curve subdivision algorithm (Chaikin algorithm) that, in the limit, produces the same curve as the uniform quadratic B-spline. [4 marks]