2010 Paper 2 Question 8

Probability

(*a*) A coin that comes up "heads" with probability p is tossed n times independently.

(*i*) What is the likelihood that k of these n tosses will be "heads", and the remainder "tails"? [2 marks]

(*ii*) Give the mean and the variance expected for the number of "heads" outcomes. [1 mark each]

(*b*) In a different experiment with this same coin, you monitor how many tosses are needed before getting the *first* outcome of a "head".

(*i*) What is the likelihood that the *first* "head" occurs on the k^{th} trial? [2 marks]

(*ii*) What is the mean trial number k for the first "head", and what is the variance for this number? [1 mark each]

(*c*) In a Poisson process with hazard parameter λ:

(*i*) What is the likelihood of observing k events? [2 marks]

(*ii*) What is the mean, and what is the variance, expected for the number of observed events? [1 mark each]

(*d*) If X and Y are random variables having expectations $E(X)$ and $E(Y)$ respectively:

(*i*) What is their covariance $Cov(X, Y)$? [2 marks]

(*ii*) In terms of their covariance $Cov(X, Y)$, and their respective variances $Var(X)$ and $Var(Y)$, what is their correlation coefficient $\rho(X, Y)$? [2 marks]

(*e*) For a continuous random variable X that is exponentially distributed, having density function $f(x) = \lambda \exp(-\lambda x)$ for $x > 0$ and $f(x) = 0$ for $x \leq 0$:

(*i*) Derive the expectation $E(X)$ of this random variable. [2 marks]

(*ii*) Derive its variance $Var(X)$. [2 marks]