2010 Paper 2 Question 5

Discrete Mathematics II

The set Ω is characterised as the least set such that

$$\text{if } X \text{ is finite and } X \subseteq \Omega\,, \text{ then } X \cup \bigcup X \in \Omega$$

[Recall $\bigcup X =_{\text{def}} \{y \mid \exists x \in X.\, y \in x\}$.]

(*a*) Show

(*i*) $\emptyset \in \Omega$, and [2 marks]

(*ii*) if $x \in \Omega$ then $\{x\} \cup x \in \Omega$. [3 marks]

(*b*) State the rule(s) and the principle of rule induction appropriate for the set Ω. [3 marks]

(*c*) Define a set x to be *transitive* iff

$$\forall z, y.\ z \in y \ \&\ y \in x \Rightarrow z \in x$$

Prove that all elements of Ω are transitive. [8 marks]

(*d*) Describe, without proof, the elements of the set Ω. [4 marks]