2010 Paper 1 Question 1

Foundations of Computer Science

(*a*) Give an ML datatype declaration suitable for representing lazy lists, possibly of infinite length.   [2 marks]

(*b*) Code the ML function `interleave`, which takes two lazy lists and generates a lazy list containing each of their elements.   [2 marks]

(*c*) Code an ML function that applies a given function to every element of a lazy list, returning a lazy list of the results (analogously to the function `map`).
                                                                                                          [3 marks]

(*d*) Code the ML function `iterates` which, given a function f and some value x, generates a lazy list containing all the values of the form $f^n(x)$ (that is, $f(\cdots f(x) \cdots)$ with n applications of f) for $n \geq 0$.   [3 marks]

(*e*) Code the ML function `iterates2` which, given functions f and g and values x and y, generates a lazy list containing all the values of the form $(f^m(x), g^n(y))$ for $m, n \geq 0$.   [10 marks]

All ML code must be explained clearly and should be free of needless complexity.