

## 2009 Paper 9 Question 15

### Types

- (a) What is meant by *beta-reduction*, *beta-conversion* and *beta-normal forms* for the polymorphic lambda calculus (PLC)? Explain why typeable PLC expressions are beta-convertible to beta-normal forms that are unique up to alpha-conversion. Is the same true for untypeable PLC expressions? (Any general properties of PLC you use should be clearly stated, but need not be proved.) [10 marks]
- (b) Let  $\tau$  be the PLC type  $\forall\beta((\alpha \rightarrow \beta) \rightarrow \beta)$ , where  $\alpha$  and  $\beta$  are distinct type variables. Give closed PLC beta-normal forms  $I$  and  $J$  with the following properties:
- (i)  $I$  has type  $\forall\alpha(\alpha \rightarrow \tau)$
  - (ii)  $J$  has type  $\forall\alpha(\tau \rightarrow \alpha)$
  - (iii)  $\Lambda\alpha(\lambda x : \alpha(J\alpha(I\alpha x)))$  has beta-normal form  $\Lambda\alpha(\lambda x : \alpha(x))$

Justify your answers by giving proofs of typing and beta-conversion.

[8 marks]

What is the beta-normal form of  $\Lambda\alpha(\lambda y : \tau(I\alpha(J\alpha y)))$ ?

[2 marks]