2009 Paper 6 Question 8

Logic and Proof

(*a*) Briefly indicate the differences between the tableau calculus and the sequent calculus. [2 marks]

(*b*) Prove the formula $[\forall x\,(P(x) \to R(x,x))] \to [\forall x\,\exists y\,(R(x,y) \vee \neg P(y))]$ using the tableau calculus, or exhibit a falsifying interpretation. [8 marks]

(*c*) Give a model for the following set of clauses, or prove that none exists. Here a is a constant, while x and y are variables. Explain your reasoning clearly.

$$\{\neg R(x,a), R(x,x)\} \qquad \{\neg R(x,x), R(x,a)\}$$

$$\{\neg R(y,f(x)), \neg R(y,x)\} \qquad \{R(y,x), R(y,f(x))\}$$

[10 marks]