2009 Paper 4 Question 10

Mathematical Methods for Computer Science

(*a*) Consider the Markov chain, X_n, on the states $i = 0, 1, 2, \ldots$ with transition matrix

$$\begin{aligned} p_{i,i-1} &= p \qquad i = 1, 2, \ldots \\ p_{i,i+1} &= 1 - p \qquad i = 0, 1, \ldots \\ p_{0,0} &= p \end{aligned}$$

where $0 < p < 1$.

(*i*) Show that the Markov chain is irreducible. [2 marks]

(*ii*) Show that the Markov chain is aperiodic. [2 marks]

(*iii*) Find a condition on p to make the Markov chain positive recurrent and find the stationary distribution in this case. [6 marks]

(*b*) Consider the PageRank algorithm.

(*i*) Describe PageRank as a Markov chain model for motion between nodes in a graph where the nodes correspond with web pages. [5 marks]

(*ii*) Explain the main mathematical results that underpin the relevance of PageRank to a notion of web page *"importance"*. [5 marks]