2009 Paper 1 Question 6

Algorithms I

(a) State the defining properties of a min-heap. Show how to convert between the tree and the (zero-based) array representation of a min-heap. [3 marks]

(b) "An array sorted in ascending order is always a min-heap." True or false? If false, offer a counter-example; otherwise, prove the correctness of this statement with respect to the defining properties of a min-heap you listed in response to part (a). [3 marks]

(c) The array

        A I E R P M S N L

is not a min-heap. Why? Redraw it as a binary tree and turn it into a heap using the $O(n)$ `heapify()` procedure normally used as part of heapsort. Draw the intermediate stages as you go along and add any necessary explanations so that a reader can follow what you are doing and why. [4 marks]

(d) Perform `extractMin()` on the min-heap you produced in part (c). As before, draw the intermediate stages and add explanations as necessary. [3 marks]

(e) What is the asymptotic running time of the heapsort algorithm on an array of length n that is already sorted in ascending order? Justify your answer. [3 marks]

(f) What is the asymptotic running time of the heapsort algorithm on an array of length n that is already sorted in *descending* order? Justify your answer. [4 marks]