2009 Paper 1 Question 5

Algorithms I

(*a*) State the *five* invariants of Red–Black Trees and briefly explain the advantages and disadvantages of Red–Black Trees over Binary Search Trees. [4 marks]

(*b*) For each of the possible types of 2–3–4-tree nodes, draw an isomorphic node cluster made of Red–Black nodes. The node clusters you produce must have the same number of keys and external links as the 2–3–4 nodes they replace and they must respect all the Red–Black tree rules when composed with other node clusters. [3 marks]

(*c*) What are the minimum and maximum possible number of nodes of a Red–Black tree with black-height h? Justify your answer. [4 marks]

(*d*) Explain, with clear pictures, what a "rotation" operation is, in the context of Binary Search Trees. [2 marks]

(*e*) Give a procedure for reshaping an arbitrary n-node Binary Search Tree containing n distinct keys into any other arbitrary Binary Search Tree with the same keys. [7 marks]