

2009 Paper 1 Question 4

Discrete Mathematics I

(a) Define what it means for a relation $R \subseteq A \times A$ to be:

(i) irreflexive [1 mark]

(ii) symmetric [1 mark]

(iii) antisymmetric [1 mark]

(b) Define a non-trivial irreflexive symmetric relation R over the set of natural numbers, showing why it has those properties. [3 marks]

(c) If A is a finite set with n elements, how many distinct irreflexive symmetric relations over A are there? [3 marks]

(d) If A is a finite set with n elements, how many distinct relations that are symmetric and antisymmetric over A are there? [3 marks]

(e) Suppose R and S are irreflexive symmetric relations over A . For each of the following relations, either prove that they are irreflexive and symmetric or give a counterexample.

(i) $R \cup S$

(ii) $R; S$

(iii) the relation Q defined to be

$$\{(X, Y) \mid X \subseteq A \wedge Y \subseteq A \wedge \forall x \in X. \forall y \in Y. (x, y) \in R\}$$

[8 marks]