

2008 Paper 9 Question 4

Quantum Computing

(a) The *no-cloning theorem* is a statement that is often said to show that a quantum state $|\phi\rangle$ cannot be exactly duplicated.

(i) Give a mathematically precise statement of the no-cloning theorem. [2 marks]

(ii) Give a proof of the no-cloning theorem. [4 marks]

(b) The *quantum teleportation* protocol is a means by which one party, Alice, can send a quantum state to another party, Bob, by transmitting just two classical bits, provided that the two already share an entangled 2-qubit state.

Explain how the quantum teleportation protocol works, sketching any circuit that may be used. [6 marks]

(c) The *Deutsch–Jozsa* problem assumes that we are given a function $f : \{0, 1\} \rightarrow \{0, 1\}$ in the form of a quantum black box performing a unitary operation $U_f : |ab\rangle \mapsto |a(b \oplus f(a))\rangle$.

Sketch a circuit with only one use of U_f that determines whether f is constant or balanced. Explain carefully what measurement is performed and why it gives the desired result. [8 marks]