2008 Paper 9 Question 15

Topics in Concurrency

This question assumes familiarity with the higher-order process language HOPLA, which has prefix types, function types, sum types and recursive types. Subject to suitable typings, HOPLA has transitions $t \xrightarrow{p} t'$ between closed terms t, t' and action p given by the following rules:

$$\frac{t[rec\,x\,t/x] \xrightarrow{p} t'}{rec\,x\,t \xrightarrow{p} t'} \qquad \frac{t_j \xrightarrow{p} t'}{\Sigma_{i\in I} t_i \xrightarrow{p} t'}\ j \in I$$

$$\frac{}{.t \xrightarrow{\bullet} t} \qquad \frac{u \xrightarrow{\bullet} u' \quad t[u'/x] \xrightarrow{p} t'}{[u > .x \Rightarrow t] \xrightarrow{p} t'}$$

$$\frac{t[u/x] \xrightarrow{p} t'}{\lambda x\,t \xrightarrow{u\mapsto p} t'} \qquad \frac{t \xrightarrow{u\mapsto p} t'}{t\,u \xrightarrow{p} t'} \qquad \frac{t \xrightarrow{p} t'}{a\,t \xrightarrow{a\,p} t'} \qquad \frac{t \xrightarrow{a\,p} t'}{\pi_a(t) \xrightarrow{p} t'}$$

(a) Let t be a term of type $\mathbb{P}$ with one free variable y of type $\mathbb{Q}$. Say t is *linear in* y iff for any sum of closed terms $\Sigma_{i\in I} u_i$ of type $\mathbb{Q}$

$$t[\Sigma_{i\in I} u_i/y] \sim \Sigma_{i\in I} t[u_i/y]\ .$$

(The relation $\sim$ is the bisimilarity congruence of HOPLA.)

Show from the transition semantics that the terms

$$\pi_a(y) \quad \text{and} \quad [y > .x \Rightarrow u] \text{ where } y \text{ is not free in } u,$$

assumed well-typed and to have only y as free variable, are linear in y.

[4 marks]

(b) For u of sum type, let $[u > a.x \Rightarrow t]$ abbreviate $[\pi_a(u) > .x \Rightarrow t]$.

Why is $[y > a.x \Rightarrow t]$ linear in y, where y is not free in t? [2 marks]

Derive a rule for the transitions of $[u > a.x \Rightarrow t]$. [2 marks]

Show $[a.u > a.x \Rightarrow t] \sim t[u/x]$ and $[b.u > a.x \Rightarrow t] \sim nil$ if $b \neq a$.
(The term *nil* represents the empty sum.) [4 marks]

(c) Describe the type you would use to interpret CCS in HOPLA. [2 marks]

Write down a HOPLA term that realises the parallel composition of CCS. What is its type? [2 marks]

State the expansion law for CCS parallel composition. In a few sentences, indicate how, using part (b), you would derive the expansion law from your definition of parallel composition. [4 marks]