2008 Paper 8 Question 9

Information Theory and Coding

(*a*) (*i*) A variable-length, uniquely decodable code that has the prefix property and whose N binary code word lengths are $n_1 \leq n_2 \leq n_3 \leq \cdots \leq n_N$ must satisfy what condition with these code word lengths? (Give an expression for the condition, and its name, but do not attempt to prove it.)
[3 marks]

(*ii*) Construct an efficient, uniquely decodable binary code, having the prefix property and having the shortest possible average code length per symbol, for an alphabet whose five letters appear with these probabilities:

Letter	A	B	C	D	E
Probability	1/2	1/4	1/8	1/16	1/16

[3 marks]

(*iii*) How do you know that, on average, for samples drawn from this alphabet, your code uses the shortest possible code length per symbol? Demonstrate numerically that your code satisfies this optimality condition. [3 marks]

(*b*) (*i*) Explain how autocorrelation can remove noise from a signal that is buried in noise, recovering a clean signal. For what kinds of signals, and for what kinds of noise, will this work best, and why? What class of signals can be recovered perfectly by autocorrelation? Begin your answer by writing down the integral that defines the autocorrelation of a signal $f(x)$.
[3 marks]

(*ii*) Some sources of noise are additive (the noise is just superimposed onto the signal), but other sources of noise are multiplicative in their effect on the signal. For which type would the autocorrelation clean-up strategy be more effective, and why? In the case of additive noise where noise and signal occupy different frequency bands, what other strategy could allow recovery of a clean signal? [3 marks]

(*c*) (*i*) If a continuous signal $f(t)$ is *modulated* by multiplying it with a complex exponential wave $\exp(i\omega t)$ whose frequency is ω, what happens to the Fourier spectrum of the signal? Name a very important practical application of this principle, and explain why modulation is a useful operation. How can *demodulation* then recover the original signal?
[3 marks]

(*ii*) Which part of the 2D Fourier Transform of an image, the amplitude spectrum or the phase spectrum, is indispensable in order for the image to be intelligible? Describe a demonstration that proves this. [2 marks]