

2008 Paper 6 Question 11

Semantics of Programming Languages

Below is the syntax and type system for a simple functional language.

Integers $n \in \mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$

Variables $x \in \mathbb{X} = \{x, y, z, \dots\}$

Types $T ::= \text{int} \mid T \rightarrow T$

Type environments Γ , finite partial functions from variables to types.

Expressions $e ::= n \mid x \mid \mathbf{fn} \ x:T \Rightarrow e \mid e_1 \ e_2$

(int) $\Gamma \vdash n:\text{int}$ for $n \in \mathbb{Z}$ (var) $\Gamma \vdash x:T$ if $\Gamma(x) = T$

(fn) $\frac{\Gamma, x:T \vdash e:T'}{\Gamma \vdash \mathbf{fn} \ x:T \Rightarrow e : T \rightarrow T'}$ (app) $\frac{\Gamma \vdash e_1:T \rightarrow T' \quad \Gamma \vdash e_2:T}{\Gamma \vdash e_1 \ e_2:T'}$

- (a) Give a call-by-value operational semantics for this language, defining a judgement $e \longrightarrow e'$ (the language does not have store operations, so you can take configurations to be just expressions). [4 marks]
- (b) Give an example of a stuck configuration. [1 mark]
- (c) Prove the substitution lemma stated below:

If $\Gamma, x:T \vdash e':T'$ and $\Gamma \vdash e:T$ with $x \notin \text{dom}(\Gamma)$ then $\Gamma \vdash \{e/x\}e':T'$.

[9 marks]

- (d) State and prove type preservation, using this substitution lemma. [6 marks]