

2008 Paper 5 Question 10

Computation Theory

(a) *The Halting Problem for register machines is unsolvable.* State, without proof, a precise form of this result. [3 marks]

(b) Let the computation by program c on data d be represented by the natural number k that codes the pair (c, d) . By considering the set $H(k)$ of the HALTING computations represented by codes $k' < k$, show that there is an increasing total function $h(k)$ which *grows too fast* to be computable. [6 marks]

(c) Given $h : \mathbb{N} \rightarrow \mathbb{N}$ with the above property

$$\begin{aligned} \text{let } f(k) &= h(k) + k \\ \text{and } g(x) &= \sup\{k : f(k) \leq x\}. \end{aligned}$$

Then $f : \mathbb{N} \rightarrow \mathbb{N}$ is strictly increasing, and $g : \mathbb{N} \rightarrow \mathbb{N}$ satisfies

$$g(f(k)) = k, \quad g(x) < k \quad \text{for all } x < f(k).$$

Show that g *grows too slowly* to be computable in the following sense:

given $G : \mathbb{N} \rightarrow \mathbb{N}$ such that

- (i) $\{G(n) : n \in \mathbb{N}\}$ is unbounded
- (ii) $G(n) \leq g(n)$ for all $n \in \mathbb{N}$

then $G(n)$ is *not* computable.

[11 marks]