2008 Paper 2 Question 3

Discrete Mathematics

Let X and Y be sets. You are reminded that a relation from X to Y is a subset of the product $X \times Y$.

(a) Explain what it means for a relation f from X to Y to be a *function*, an *injection* and a *surjection* from X to Y. [4 marks]

(b) A *bijection* from X to Y is defined to be a function from X to Y which is both an injection and a surjection. Prove that a function f from X to Y is a bijection iff it has an inverse function g, i.e. g is a function from Y to X such that $g \circ f = id_X$ and $f \circ g = id_Y$.
[Remember to prove both the "if" and "only if" parts of the assertion.]
[12 marks]

(c) Describe, without proof, a bijection from $\mathcal{P}(X \times Y)$ to $(X \to \mathcal{P}(Y))$ and its inverse. [4 marks]