# 2007 Paper 9 Question 14

## Types

(*a*) Give the rules inductively defining the type system of the *polymorphic lambda calculus* (PLC). [5 marks]

(*b*) What does it mean for a PLC expression $M$ to be in *beta-normal form*? [2 marks]

(*c*) The *long normal forms*, $L$, and the *neutral forms*, $N$, are special kinds of PLC expression given by the following grammar:

$$\begin{aligned} L &::= \lambda x : \tau(L) \mid \Lambda\alpha(L) \mid N, \\ N &::= x \mid N\,L \mid N\,\tau. \end{aligned}$$

(*i*) Arguing by induction on the structure of $L$ and $N$, or otherwise, show that all such expressions are in beta-normal form. [4 marks]

(*ii*) Show that if $N$ is a neutral form, then $\{\} \vdash N : \tau$ is not provable in the PLC type system for any type $\tau$, where $\{\}$ is the empty typing environment. (You may assume without proof that if $\Gamma \vdash M : \tau$ is provable in the PLC type system, then the free variables of the expression $M$ are contained in the domain of definition of the typing environment $\Gamma$.) [3 marks]

(*iii*) Hence prove that for any long normal form $L$, $\{\} \vdash L : \forall\alpha(\alpha)$ is not provable in the PLC type system. [6 marks]