Quantum Computing

(a) Consider a quantum finite automaton with two basis states, $|0\rangle$ being the start state and $|1\rangle$ the only accepting state. The automaton operates on a two-letter alphabet, with matrices $M_a = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ and $M_b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

What are the probabilities that the automaton accepts each of the following input strings?

(i) $a$ [3 marks]

(ii) $aba$ [3 marks]

(iii) $abb$ [3 marks]

(b) Give a complete description of the probabilities of acceptance associated with various possible input strings. [5 marks]

(c) Prove that there is no two-state probabilistic automaton with the same behaviour as the automaton described in part (a). [6 marks]