

2007 Paper 6 Question 9

Logic and Proof

For *each* of the following formulae, state (with justification) whether it is satisfiable, valid or neither:

$$\begin{aligned} & ((P \vee Q) \rightarrow R) \leftrightarrow (P \rightarrow (Q \rightarrow R)) \\ & (P \wedge \neg Q) \vee (\neg R \wedge Q) \vee (R \wedge \neg P) \vee (\neg P \wedge \neg Q \wedge \neg R) \vee (P \wedge Q \wedge R) \\ & \left[\forall x \exists y (P(x) \rightarrow Q(x, y)) \wedge \forall x \exists z \forall y (\neg Q(x, y) \rightarrow P(z)) \right] \rightarrow \exists x \forall y Q(x, y) \end{aligned}$$

[4+7+9 marks]