2007 Paper 5 Question 12

Complexity Theory

(*a*) Give a precise definition of polynomial-time reductions. [2 marks]

(*b*) Give a precise definition of NP-completeness. [3 marks]

(*c*) Let **Subset Sum** denote the following decision problem:

Given a set of positive integers $S = \{v_1, \ldots, v_n\}$ and a number t, determine whether there is a subset of S that sums to exactly t.

(*i*) Explain why **Subset Sum** is in NP. [3 marks]

(*ii*) Describe a polynomial-time reduction from the problem of 3-dimensional matching to **Subset Sum**. [9 marks]

(*iii*) Explain why parts (*i*) and (*ii*) above imply that **Subset Sum** is NP-complete. [3 marks]