

## 2007 Paper 3 Question 7

### Computation Theory

- (a) (i) Define the notion of a *register machine* and the computations that it carries out. [5 marks]
- (ii) Explain, in general terms, what is meant by a *universal* register machine. (You should make clear what scheme for coding programs as numbers you are using, but you are not required to describe a universal register machine program in detail.) [5 marks]
- (b) (i) Explain what it means for a partial function  $f$  from  $\mathbb{N}$  to  $\mathbb{N}$  to be *computable* by a register machine. [2 marks]
- (ii) Let  $n > 1$  be a fixed natural number. Show that the partial function from  $\mathbb{N}$  to  $\mathbb{N}$
- $$f_n(x) = \begin{cases} nx & \text{if } x > 0 \\ \text{undefined} & \text{if } x = 0 \end{cases}$$
- is computable. [3 marks]
- (iii) Explain why there are only countably many computable functions from  $\mathbb{N}$  to  $\mathbb{N}$ . Deduce that there exists a partial function from  $\mathbb{N}$  to  $\mathbb{N}$  that is not computable. (Any standard results you use about countable and uncountable sets should be clearly stated, but need not be proved.) [3 marks]
- (iv) If a partial function  $f$  from  $\mathbb{N}$  to  $\mathbb{N}$  is computable, how many different register machine programs are there that compute  $f$ ? [2 marks]