2007 Paper 11 Question 9

Data Structures and Algorithms

(a) Without dwelling on the structure of the nodes and on the positional relationship between keys and subtrees (for which an example picture will be sufficient), give an otherwise complete and concise definition of a B-tree of minimum degree t, listing all the defining structural properties. [2 marks]

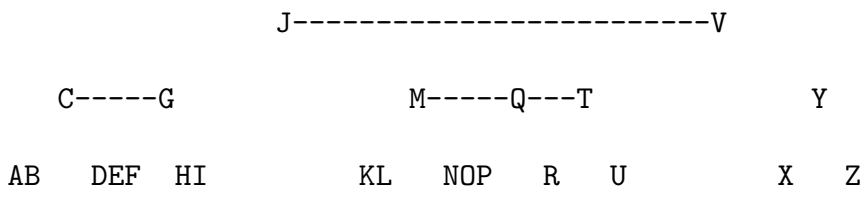
(b) Explain how to insert a new item into a B-tree, showing that the algorithm preserves the B-tree properties you gave in part (a). Then insert the following values, in this order, into an initially empty B-tree whose nodes hold at most three keys each: `C A M B R I D G E X`. Produce a frame-by-frame "movie" in which you redraw the tree whenever it changes in any way. [4 marks]

(c) Define a "bottom node" as any internal node whose children are (keyless) leaves. Prove that, for any key in any node of a B-tree, either the key is in a bottom node or its successor is. You may, for simplicity, assume that all keys are distinct. [4 marks]

(d) Explain how to delete a value from a B-tree:

(i) Explain the overall strategy, with diagrams and pseudocode where helpful, convincing the reader that it covers all possible cases and preserves the B-tree properties.

(ii) Apply this procedure to the deletion of values M, Q, Y, in that order, from the following B-tree, producing a frame-by-frame movie as requested for part (b). As before, each node holds at most three keys.



[10 marks]