2007 Paper 11 Question 6

Computation Theory

(*a*) What does it mean for a set of natural numbers $S \subseteq \mathbb{N}$ to be

    (*i*) *recursive*? [1 mark]

    (*ii*) *recursively enumerable*? [2 marks]

(*b*) Show that if a set is recursive, then it is also recursively enumerable. [5 marks]

(*c*) Let ϕ_e denote the partial function from $\mathbb{N}$ to $\mathbb{N}$ computed by the register machine with code $e \in \mathbb{N}$. Is either of the following sets of numbers recursively enumerable? Justify your answer in each case, stating clearly any standard results that you use.

    (*i*) $S_1 = \{e \in \mathbb{N} \mid \text{for all } x \in \mathbb{N},\ \phi_e(x) \text{ is defined}\}$. [6 marks]

    (*ii*) $S_2 = \{e \in \mathbb{N} \mid \text{for some } x \in \mathbb{N},\ \phi_e(x) \text{ is defined}\}$. [6 marks]